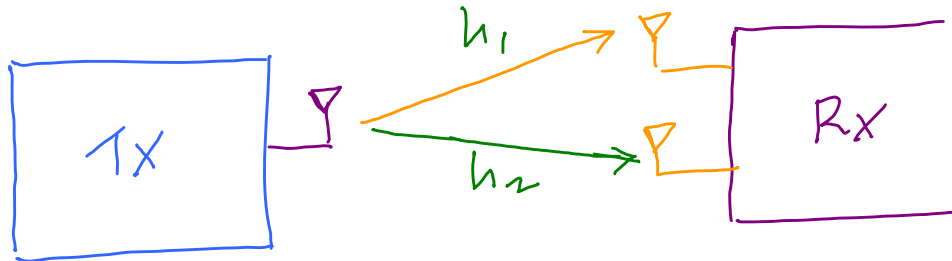


Example:



$$h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$$

$$h_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$$

$$y_1 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \right) x + n_1$$

$$y_2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \right) x + n_2$$

$$E\{|n_1|^2\} = E\{|n_2|^2\} = \sigma^2 = \frac{1}{2}$$

$$10 \log_{10} \frac{1}{2} = -3 \text{ dB.}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}}_{\bar{h}} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\bar{y} = \bar{h} x + \bar{n}$$

$$\text{Optimal MRC vector} = \frac{\bar{h}}{\|\bar{h}\|}$$

$$\|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2}$$

$$\vec{h} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$$

$$\|\vec{h}\| = \sqrt{|h_1|^2 + |h_2|^2}$$

$$= \sqrt{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$$

$$= \sqrt{2} \quad \boxed{\frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}}$$

Maximal
Ratio
Combining
vector

$$\vec{w} = \frac{\vec{h}}{\|\vec{h}\|} =$$

$$SNR = \frac{|\bar{w}^H \bar{h}|^2 P}{\sigma^2 \|\bar{w}\|^2}$$

$$\bar{w}^H \bar{h} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right]$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$|\bar{w}^H \bar{h}|^2 = (\sqrt{2})^2 = 2$$

$$\|\bar{W}\|^2 = |W_1|^2 + |W_2|^2$$

$$= \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right)$$

$$= \frac{1}{2} \cdot 2 = 1$$

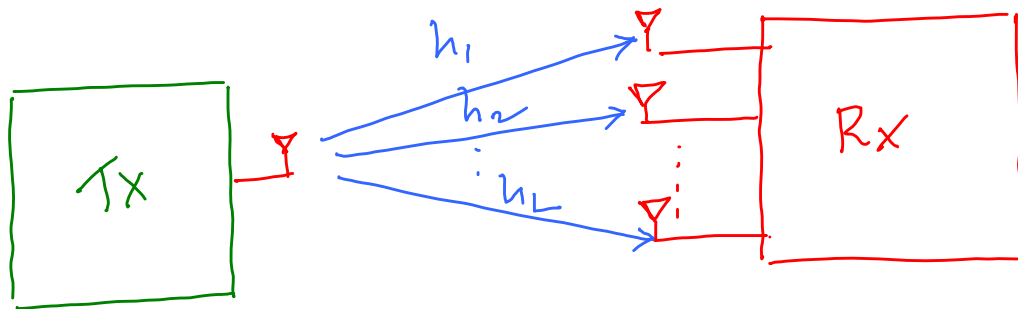
$$SNR = \frac{P \cdot |\bar{W}^H \mathbf{h}|^2}{\sigma^2 \|\bar{W}\|^2} = \frac{P \cdot 2}{\frac{1}{2} \cdot 1}$$

$$= 4P.$$

$$\boxed{SNR = 4P}$$

← For MRC

Generalize MRC to L Antennas:



channel coefficients = $h_1, h_2, \dots, h_L$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

$$\bar{y} = \bar{h}x + \bar{n} \quad \text{Noise power} = \sigma^2$$

$$E\{|n_i|^2\} = \sigma^2$$

noise on
any pair of
antennas is
uncorrelated

$$E\{n_i n_j\} = 0 \quad \forall i \neq j$$

$$\bar{y} = W_1^* y_1 + W_2^* y_2 + \dots + W_L^* y_L$$

$$\begin{aligned}
 &= \bar{\mathbf{w}}^H \bar{\mathbf{y}} = \bar{\mathbf{w}}^H (\bar{\mathbf{h}} x + \bar{\mathbf{n}}) \\
 &= \underbrace{\bar{\mathbf{w}}^H \bar{\mathbf{h}} x}_{\text{signal}} + \underbrace{\bar{\mathbf{w}}^H \bar{\mathbf{n}}}_{\text{noise}}
 \end{aligned}$$

For maximum SNR, choose
MRC (maximal ratio combining)

$$\bar{\mathbf{w}} = \frac{\bar{\mathbf{h}}}{\|\bar{\mathbf{h}}\|} = \frac{1}{\|\bar{\mathbf{h}}\|} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix}$$

$$\|h\| = \sqrt{|h_1|^2 + |h_2|^2 + \dots + |h_L|^2}$$

SNR with maximal Ratio Combining

$$= \boxed{\frac{\|h\|^2 P}{\sigma^2}}$$

$$= \frac{P(|h_1|^2 + |h_2|^2 + \dots + |h_L|^2)}{\sigma^2}$$